

# Phase structure of the QCD vacuum in a magnetic field at low temperature

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## Abstract

We study the QCD phase structure in magnetic field  $H$  at low temperature  $T$ . The hadronic phase free energy in a constant homogeneous magnetic field is calculated in one-loop approximation of the chiral perturbation theory. The dependence of the quark and gluon condensates upon the temperature and field strength is found. It is shown that the chiral phase transition order parameter  $\langle \bar{q}q \rangle$  remains constant provided field strength and temperature are related via  $H = 6.421T^2$ .

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1. The investigation of the vacuum state behavior under the influence of the various external factors is known to be one of the central problems in quantum field theory. In the realm of strong interactions (QCD) the main factors are the temperature and the baryon density. At temperatures below the chiral phase transition,  $T < T_c$ , the dynamics of the system is characterized by confinement and spontaneous breaking of chiral symmetry (SBCS). At low temperatures,  $T < T_c$ , the partition function of the system is dominated by the contribution of the lightest particles in the physical spectrum. In QCD this role is played by the  $\pi$  – meson which is the Goldstone excitation mode in chiral condensate. Therefore the low temperature physics (the hadron phase) enables an adequate description in terms of the effective chiral theory [1, 2, 3]. A very important problem is the behavior of the order parameter (the quark condensate  $\langle \bar{q}q \rangle$ ) with the increase of the temperature. In the ideal gas approximation the contribution of the thermal pions into the quark condensate is proportional to  $T^2$  [4, 5]. In chiral perturbation theory (ChPT) the two – and three–loops contributions ( $\sim T^4$  and  $\sim T^6$  correspondingly) into  $\langle \bar{q}q \rangle$  have been found in [5] and [6].

The situation with the gluon condensate  $\langle G^2 \rangle \equiv \langle (gG_{\mu\nu}^a)^2 \rangle$  is very different. The gluon condensate is not an order parameter in phase transition and it does not lead to any spontaneous symmetry breaking (SSB). At the quantum level the trace anomaly leads to the breaking of the scale invariance and this in turn results in nonzero value of  $\langle G^2 \rangle$ . However, this is not a SSB phenomenon and hence does not lead to the appearance of the Goldstone particle. The mass of the lowest excitation (dilaton) is directly connected to the gluon condensate,  $m_D \propto (\langle G^2 \rangle)^{1/4}$ . Thus the thermal excitations of glueballs are exponentially suppressed by the Boltzmann factor  $\sim \exp\{-m_{gl}/T\}$  and their contribution to the shift of the gluon condensate is small ( $\Delta\langle G^2 \rangle/\langle G^2 \rangle \sim 0.1\%$  at  $T = 100$  MeV) [7]. Next we note that in the one-loop approximation ChPT pions are described as a gas of massless noninteracting particles. Such a system is obviously scale-invariant and therefore does not contribute into the trace of the energy-momentum tensor and correspondingly into  $\langle G^2 \rangle$ . As it has been demonstrated in [8] the gluon condensate temperature dependence arises only at the ChPT three–loop level.

Another interesting problem is the study of the vacuum phase structure under the influence of the external magnetic field  $H$ . The behavior of the quark condensate in the presence of a magnetic field was studied in Nambu-Lasinio model earlier [9]. For QCD, the analogous investigation in the one-loop approximations was done in [10]. It was found that the quark con-

condensate grows with the increase of the magnetic field  $H$  in both cases. It implies that a naive analogy with superconductivity, where the order parameter vanishes at same critical field, is not valid here. The behavior of the gluon condensate  $\langle G^2 \rangle$  in the Abelian magnetic field is also a nontrivial effect. Gluons do not carry electric charge; nevertheless, virtual quarks produced by them interact with electromagnetic field and lead to the changes in the gluon condensate. This phenomenon was studied in [11], [12] based on the low-energy theorems in QCD. The vacuum energy density, the values of  $\langle G^2 \rangle$  and  $\langle \bar{q}q \rangle$  as functions of  $H$  have been found in the two-loop approximation ChPT in [12].

The low-energy theorems, playing an important role in the understanding of the vacuum state properties in quantum field theory, were discovered almost at the same time as quantum field methods appeared in particle physics (see, for example Low theorems [13]). In QCD, these theorems were obtained in the beginning of eighties [14]. These theorems, being derived from the very general symmetrical considerations and not depending on the details of confinement mechanism, sometimes gives information which is not easy to obtain in another way. Also, they can be used as "physically sensible" restrictions in the constructing of effective theories. An important step was made in [15], where low-energy theorems for gluodynamics were generalized to finite temperature case.

In the present work the vacuum free energy in magnetic field at finite temperature is calculated in the framework of ChPT. The general relations are established which allow to obtain the dependence of the quark and gluon condensates on  $T$  and  $H$ . A new phenomenon is displayed, namely the "freezing" of the chiral phase transition order parameter by the magnetic field when the temperature increases. The physical meaning of this fact is discussed.

2. The QCD Euclidean partition function in external Abelian field  $A_\mu$  has the following form ( $T = 1/\beta$ )

$$Z = \exp \left\{ -\frac{1}{4e^2} \int_0^\beta dx_4 \int_V d^3x F_{\mu\nu}^2 \right\} \int [DB][D\bar{q}][Dq] \exp \left\{ -\int_0^\beta dx_4 \int_V d^3x \mathcal{L} \right\}. \quad (1)$$

Here the QCD Lagrangian in the background field is

$$\mathcal{L} = \frac{1}{4g_0^2} (G_{\mu\nu}^a)^2 + \sum_{q=u,d} \bar{q} [\gamma_\mu (\partial_\mu - iQ_q A_\mu - i\frac{\lambda^a}{2} B_\mu^a) + m_q] q, \quad (2)$$

where  $Q_q$  – is the matrix of the quark charges for the quarks  $q = (u, d)$ , and for the simplicity the ghost terms have been omitted. The free energy density is given by the relation  $\beta V F(T, H, m_q) = -\ln Z$ . In the chiral limit  $m_q \rightarrow 0$  Eq. (1) yields the following expressions for the quark and gluon condensates

$$\langle \bar{q}q \rangle(T, H) = \frac{\partial F(H, T, m_q)}{\partial m_q} \Big|_{m_q=0}, \quad (3)$$

$$\langle G^2 \rangle(T, H) = 4 \frac{\partial F(H, T, m_q)}{\partial(1/g_0^2)} \Big|_{m_q=0}. \quad (4)$$

The phenomenon of dimensional transmutation results in the appearance of a nonperturbative dimensional parameter

$$\Lambda = M \exp \left\{ \int_{\alpha_s(M)}^{\infty} \frac{d\alpha_s}{\beta(\alpha_s)} \right\}, \quad (5)$$

where  $M$  is the ultraviolet cutoff,  $\alpha_s = g_0^2/4\pi$ , and  $\beta(\alpha_s) = d\alpha_s(M)/d \ln M$  is the Gell-Mann-Low function. In chiral limit ( $m_q = 0$ ) the system described by the partition function (1) is characterized by three dimensionful parameters  $M, T$  and  $H$ . The free energy density is renorm-invariant quantity and hence its anomalous dimension is zero. Thus  $F$  has only a normal (canonical) dimension equal to 4. Making use of the renorm-invariance of  $\Lambda$ , one can write in the most general form

$$F = \Lambda^4 f\left(\frac{T}{\Lambda}, \frac{H}{\Lambda^2}\right), \quad (6)$$

where the function  $f$  is still unknown. From (5) and (6) one gets

$$\frac{\partial F}{\partial(1/g_0^2)} = \frac{\partial F}{\partial \Lambda^2} \frac{\partial \Lambda^2}{\partial(1/g_0^2)} = \frac{4\pi\alpha_s^2}{\beta(\alpha_s)} \left(4 - T \frac{\partial}{\partial T} - 2H \frac{\partial}{\partial H}\right) F. \quad (7)$$

With the account of (4) the gluon condensate is given by

$$\langle G^2 \rangle(T, H) = \frac{16\pi\alpha_s^2}{\beta(\alpha_s)} \left(4 - T \frac{\partial}{\partial T} - 2H \frac{\partial}{\partial H}\right) F(T, H) \quad (8)$$

At  $T = 0, H = 0$  we recover the well known expression for the nonperturbative vacuum energy density in the chiral limit. In the one-loop approximation ( $\beta = -b_0\alpha_s^2/2\pi, b_0 = (11N_c - 2N_f)/3$ ) it has the form

$$\varepsilon_v = F(0, 0) = -\frac{b_0}{128\pi^2} \langle G^2 \rangle \quad (9)$$

Let us now derive the low-energy theorems at finite temperature in the presence of magnetic field. Strictly speaking,  $\beta$ -function depends on  $H$  and the low-energy theorems acquire electromagnetic corrections  $\propto e^4$ . However, since the free energy is independent of the scale  $M$  at which the ultraviolet divergences are regulated, we can choose  $M^2 \gg H, T^2, \Lambda^2$ . Hence, we can take the lowest order expression for  $\beta$ -function ( $\beta(\alpha_s) = -b_0\alpha_s^2/2\pi$ ) and the electromagnetic corrections vanish. Let us introduce the field  $\sigma(\tau = x_4, \mathbf{x})$  and operator  $\hat{D}$ ,

$$\sigma(\tau, \mathbf{x}) = -\theta_{\mu\mu}(\tau, \mathbf{x}) = \frac{b_0}{32\pi^2}(G_{\mu\nu}^a(\tau, \mathbf{x}))^2, \quad (10)$$

$$\hat{D} = 4 - T \frac{\partial}{\partial T} - 2H \frac{\partial}{\partial H}. \quad (11)$$

Differentiating (4)  $n$  times with respect to  $(1/g_0^2)$  and taking into account (7), (10) and (11) one obtains

$$\begin{aligned} \hat{D}^{n+1}F &= (-1)^n \hat{D}^n \langle \sigma(0, \mathbf{0}) \rangle \\ &= \int d\tau_n d^3x_n \dots \int d\tau_1 d^3x_1 \langle \sigma(\tau_n, \mathbf{x}_n) \dots \sigma(\tau_1, \mathbf{x}_1) \sigma(0, \mathbf{0}) \rangle_c. \end{aligned} \quad (12)$$

The subscript  $c$  means that only connected diagrams are to be included. Proceeding in the same way, it is possible to obtain the similar theorems for renorm-invariant operator  $O(x)$  which is built from quark and/or gluon fields

$$\begin{aligned} &\left( T \frac{\partial}{\partial T} + 2H \frac{\partial}{\partial H} - d \right)^n \langle O \rangle \\ &= \int d\tau_n d^3x_n \dots \int d\tau_1 d^3x_1 \langle \sigma(\tau_n, \mathbf{x}_n) \dots \sigma(\tau_1, \mathbf{x}_1) O(0, \mathbf{0}) \rangle_c. \end{aligned} \quad (13)$$

where  $d$  is the canonical dimension of operator  $O$ . If operator  $O$  has also anomalous dimension, the appropriate  $\gamma$ -function should be included.

3. The above equations enable to obtain the values of the condensates as functions of  $T$  and  $H$  provided the free energy density is known. To get the latter the ChPT will be used. At low temperatures  $T < T_c$  ( $T_c$  is the chiral phase transition temperature) and for weak fields  $H < \mu_{had}^2 \sim (4\pi F_\pi)^2$  the characteristic momenta in the vacuum loops are small and theory is adequately described by the low-energy effective chiral Lagrangian  $L_{eff}$  [2, 3].

This Lagrangian can be represented as a series expansion over the momenta (derivatives) and quark masses

$$L_{eff} = L^{(2)} + L^{(4)} + L^{(6)} + \dots \quad (14)$$

The leading term in (14) is similar to the Lagrangian of the non-linear sigma model in the external field

$$L^{(2)} = \frac{F_\pi^2}{4} \text{Tr}(\nabla_\mu U^\dagger \nabla_\mu U) + \Sigma \text{Re Tr}(\hat{M}U^\dagger), \quad \nabla_\mu U = \partial_\mu U - i[U, V_\mu]. \quad (15)$$

Here  $U$  is a unitary  $SU(2)$  matrix,  $F_\pi = 93\text{MeV}$  is the pion decay constant, and  $\Sigma$  has the meaning of the quark condensate  $\Sigma = |\langle \bar{u}u \rangle| = |\langle \bar{d}d \rangle|$ . The external Abelian magnetic field  $H$  is aligned along the  $z$ -axis and corresponds to  $V_\mu(x) = (\tau^3/2)A_\mu(x)$  with the vector-potential  $A_\mu$  chosen as  $A_\mu(x) = \delta_{\mu 2} H x_1$ . The mass difference between the  $u$  and  $d$  quarks appears in the effective chiral Lagrangian only quadratically. Further, to obtain an expression for the quark condensate in the chiral limit we use only the first derivative with respect to the mass of one of the quarks. Therefore, we can neglect the mass difference between the  $u$  and  $d$  quarks and assume the mass matrix to be diagonal  $\hat{M} = m\hat{I}$ .

At  $T < T_c, H < \mu_{had}^2$  the QCD partition function coincides with the partition function of the effective chiral theory

$$Z_{eff}[T, H] = e^{-\beta V F_{eff}[T, H]} = Z_0[H] \int [DU] \exp\left\{-\int_0^\beta dx_4 \int_V d^3x L_{eff}[U, A]\right\} \quad (16)$$

At the one-loop level it is sufficient to restrict the expansion of  $L_{eff}$  by the quadratic terms with respect to the pion field. Using the exponential parameterization of the matrix  $U(x) = \exp\{i\tau^a \pi^a(x)/F_\pi\}$  one finds

$$L^{(2)} = \frac{1}{2}(\partial_\mu \pi^0)^2 + \frac{1}{2}M_\pi^2(\pi^0)^2 + (\partial_\mu \pi^+ + iA_\mu \pi^+)(\partial_\mu \pi^- - iA_\mu \pi^-) + M_\pi^2 \pi^+ \pi^-, \quad (17)$$

where the charged  $\pi^\pm$  and neutral  $\pi^0$  meson fields are introduced

$$\pi^\pm = (\pi^1 \pm i\pi^2)/\sqrt{2}, \quad \pi^0 = \pi^3 \quad (18)$$

Thus (16) can be recasted into the form

$$Z_{eff}^R[T, H] = Z_{p.t.}^{-1} Z_0[H] \int [D\pi^0][D\pi^+][D\pi^-] \exp\left\{-\int_0^\beta dx_4 \int_V d^3x L^{(2)}[\pi, A]\right\} \quad (19)$$

where partition function is normalized for the case of perturbation theory at  $T = 0, H = 0$

$$Z_{p.t.} = [\det(-\partial_\mu^2 + M_\pi^2)]^{-3/2}. \quad (20)$$

Integration of (19) over  $\pi$ -fields leads to

$$Z_{eff}^R = Z_{p.t.}^{-1} Z_0[H] [\det_T(-\partial_\mu^2 + M_\pi^2)]^{-1/2} [\det_T(-|D_\mu|^2 + M_\pi^2)]^{-1}, \quad (21)$$

where  $D_\mu = \partial_\mu - iA_\mu$  is a covariant derivative and a symbol  $T$  means that the determinant is calculated at finite temperature  $T$  according to standard Matsubara rules. Taking (20) into account and regrouping multipliers in (21) one gets the following expression for  $Z_{eff}^R$

$$\begin{aligned} Z_{eff}^R[T, H] = Z_0[H] & \left[ \frac{\det_T(-\partial_\mu^2 + M_\pi^2)}{\det(-\partial_\mu^2 + M_\pi^2)} \right]^{-1/2} \left[ \frac{\det(-|D_\mu|^2 + M_\pi^2)}{\det(-\partial_\mu^2 + M_\pi^2)} \right]^{-1} \\ & \times \left[ \frac{\det_T(-|D_\mu|^2 + M_\pi^2)}{\det(-|D_\mu|^2 + M_\pi^2)} \right]^{-1} \end{aligned} \quad (22)$$

Then the effective free energy can be written in the form

$$F_{eff}^R(T, H) = -\frac{1}{\beta V} \ln Z_{eff}^R = \frac{H^2}{2e^2} + F_{\pi^0}(T) + F_{\pi^\pm}(H) + F_s(T, H). \quad (23)$$

Here  $F_{\pi^0}$  is the free energy of massive scalar boson

$$F_{\pi^0}(T) = T \int \frac{d^3p}{(2\pi)^3} \ln(1 - \exp(-\sqrt{\mathbf{p}^2 + M_\pi^2}/T)), \quad (24)$$

$F_{\pi^\pm}$  is a Schwinger result for the vacuum energy density of charged scalar particles in the magnetic field.

$$F_{\pi^\pm}(H) = -\frac{1}{16\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-M_\pi^2 s} \left[ \frac{Hs}{\sinh(Hs)} - 1 \right], \quad (25)$$

and

$$\begin{aligned} F_s(T, H) &= \frac{HT}{\pi^2} \sum_{n=0}^\infty \int_0^\infty dk \ln(1 - \exp(-\omega_n/T)), \\ \omega_n &= \sqrt{k^2 + M_\pi^2 + H(2n+1)}, \end{aligned} \quad (26)$$

where  $\omega_n$  are Landau levels of the  $\pi^\pm$  mesons in constant field  $H$ .

4. The free energy  $F_{eff}^R$  determines the thermodynamical properties and the phase structure of the QCD vacuum state below the temperature of the chiral phase transition, i.e. in the phase of confinement.

Equations (8) and (23) describe the dependence of  $\langle G^2 \rangle$  on  $T$  and  $H$ . The action of the operator  $\hat{D}$  on  $F_{eff}^R$  leads to  $\hat{D}F_{\pi^0}(T) = 0$  since  $M_\pi^2 \rightarrow 0$  and  $F_{\pi^0}(T) \sim T^4$  in chiral limit and thus  $(4-T\partial/\partial T)F_{\pi^0}(T) = 0$ . It can be easily shown by direct calculation that  $\hat{D}F_s(T, H) = 0$ . The nontrivial dependence of  $\langle G^2 \rangle$  on  $H$  arises only due to Schwinger term  $F_{\pi^\pm}(H)$

$$\langle G^2 \rangle(T, H) = \langle G^2 \rangle + \frac{\alpha_s^2}{3\pi\beta(\alpha_s)} H^2 \quad (27)$$

Next we note that because of the asymptotic freedom the QCD  $\beta$ -function,  $\beta(\alpha_s) = -b_0\alpha_s^2/2\pi + \dots$  is negative and hence the gluon condensate diminishes with the  $H$  increasing

$$\langle G^2 \rangle(T, H) = \langle G^2 \rangle - \frac{2}{3b_0} H^2.$$

In order to get the dependence of the quark condensate upon  $T$  and  $H$  use is made of the Gell-Mann-Oakes-Renner relation (GMOR)

$$F_\pi^2 M_\pi^2 = 2m\Sigma \quad (28)$$

Substituting (23) into (3), calculating the derivative over  $M_\pi^2$  and then taking the limit  $M_\pi^2 \rightarrow 0$  one gets

$$\begin{aligned} \langle \bar{q}q \rangle(T, H) &= \langle \bar{q}q \rangle \left( 1 - \frac{1}{3} \cdot \frac{T^2}{8F_\pi^2} + \frac{H}{(4\pi F_\pi)^2} \ln 2 - \frac{H}{2\pi^2 F_\pi^2} \varphi\left(\frac{\sqrt{H}}{T}\right) \right) \\ \varphi(\lambda) &= \sum_{n=0}^{\infty} \int_0^{\infty} \frac{dx}{\omega_n(x)(\exp(\lambda\omega_n(x)) - 1)}, \quad \omega_n(x) = \sqrt{x^2 + 2n + 1} \end{aligned} \quad (29)$$

Now we consider various limiting cases. When  $H \rightarrow 0, T = \text{const}$ , for the  $F_s$  term in the free energy one finds<sup>1</sup>

$$F_s(T, H \rightarrow 0) = 2F_{\pi^0}(T) + \frac{H^2}{24\pi^2} f\left(\frac{M_\pi^2}{T^2}\right),$$

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<sup>1</sup>To get (30) the sum in  $F_s$  is calculated with the needed accuracy using the Euler-MacLaren relation

$$\sum_{n=0}^{\infty} \phi(n + 1/2) = \int_0^{+\infty} dx \phi(x) + \frac{1}{24} \phi'(0)$$

Then, using the eqs. (3), (8), (23) and (30), taking the limit  $H \rightarrow 0$  and setting  $M_\pi^2 = 0$  afterwards, we arrive at (31)



$$f(\alpha) = \int_0^\infty \frac{dx}{\sqrt{x^2 + \alpha}(\exp(\sqrt{x^2 + \alpha}) - 1)} \quad (30)$$

The corresponding expressions for condensates read

$$\begin{aligned} \langle \bar{q}q \rangle(T, H \rightarrow 0) &= \langle \bar{q}q \rangle \left(1 - \frac{T^2}{8F_\pi^2}\right), \\ \langle G^2 \rangle(T, H \rightarrow 0) &= \langle G^2 \rangle. \end{aligned} \quad (31)$$

Thus at  $H \rightarrow 0, T = \text{const}$ , we arrive at the known result [4, 5, 8]. In the reverse limit  $T \rightarrow 0, H = \text{const}$ , we get [10, 11, 12]

$$\langle \bar{q}q \rangle(T \rightarrow 0, H) = \langle \bar{q}q \rangle \left(1 + \frac{H}{(4\pi F_\pi)^2} \ln 2\right), \quad (32)$$

$$\langle G^2 \rangle(T \rightarrow 0, H) = \langle G^2 \rangle + \frac{\alpha_s^2}{3\pi\beta(\alpha_s)} H^2 \quad (33)$$

An interesting phenomenon reveals itself in the vacuum QCD phase structure under consideration. One can find from (29) such a function  $H(T)$  that the chiral condensate  $\langle \bar{q}q \rangle(T, H)$  remains unchanged when the temperature and magnetic field change in accordance with  $H_* = H(T)$ . Let us introduce the new variable  $\lambda = \sqrt{H}/T$ . Then  $H_*$  is found by solving the following equation (see (29)  $\langle \bar{q}q \rangle(T, H_*) - \langle \bar{q}q \rangle = 0$ )

$$1 - (3/2)\lambda^2 \ln 2 + 12\lambda^2 \varphi(\lambda) = 0 \quad (34)$$

The numerical solution of (34) yields  $\lambda_* = 2.534\dots$ . Thus, quark condensate stays unchanged when  $T$  and  $H$  are increased according to  $H = 6.421 \cdot T^2$ . Hence it is possible to say that the order parameter  $\langle \bar{q}q \rangle$  of the chiral phase transition is "frozen" by the magnetic field.

Note that  $H(T_c)/(4\pi F_\pi)^2 \simeq 0.1 \ll 1$  at  $T = T_c \simeq 150\text{MeV}$  and therefore the above relations remain valid up to the deconfined phase transition point. In the vicinity of  $T_c$  the effective low energy chiral Lagrangian fails to provide an adequate description of the QCD vacuum thermodynamical properties, and strictly speaking becomes physically invalid.

The following is worth noting. In deriving (29-33), at the first step the physical quantities and their limiting values as functions of  $M_\pi$  were obtained, and only then the chiral limit  $M_\pi \rightarrow 0$  was taken. Acting in the inversed sequence we would have obtained all temperature corrections to

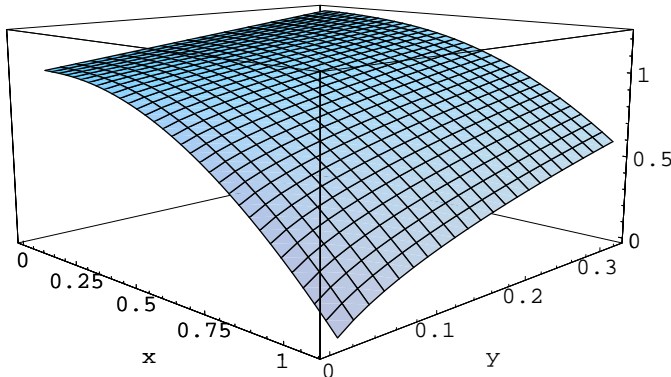


Figure 1: Quark condensate  $\Sigma(T, H)/\Sigma$  as function of  $x = T/\sqrt{8}F_\pi$  and  $y = H/(4\pi F_\pi)^2$

condensates identically equal to zero. This points to the fundamental difference of the two cases: the exactly massless particle and the particle with infinitesimal small mass.

5. It has been shown in the present work that the quark condensate is "frozen" by the magnetic field when both temperature  $T$  and magnetic field  $H$  are increased according to the  $H = \text{const} \cdot T^2$  law. This points to the fact that the direct analogy between the quark condensate in QCD and the theory of superconductivity is untenable. In the BCS theory the Cooper pairs condensates is extinguished by the temperature and magnetic field. The "freezing" phenomenon can be understood in terms of the general Le Chatelier–Braun principle<sup>2</sup>. The external field contributes into the system an additional energy density  $H^2/2$ . The system tends to compensate this energy change and to decrease the free energy by increasing the absolute value of the quark condensate:  $\Delta\varepsilon_v = -m|\Sigma(H) - \Sigma(0)| < 0$ . On the other hand, if the temperature of the system is increased (by bringing some heat into it) the processes with heat absorbtion by damping the condensate are switched on. The interplay of these processes is at the origin of the above "freezing" of  $\Sigma(T, H)$ . Next, since gluons do not carry electric charge, the magnetic field affects the gluon sector of the vacuum only indirectly via the quark sector and thus the Le Chatelier–Braun principle is not applicable directly to the gluon condensate. For the same reason gluon condensate decreases

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<sup>2</sup>The external action disturbing the system from the equilibrium state induces processes in this system which tend to reduce the result of this action

nonlinearly with  $H$  increasing according to  $\Delta\langle G^2\rangle \propto -H^2$ , while for the quark condensate  $\Delta\Sigma \propto H$ .

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